2021

COMPUTER SCIENCE — HONOURS

Third Paper

Full Marks : 100

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** from the rest, taking at least ***two*** from **Group-A** and at least ***one*** from **Group-B** and **Group-C**.

1. Answer ***any ten*** questions : 2×10

(a) When is a graph called a universal graph? Give a suitable example.

(b) What is a spanning tree of a connected graph?

(c) Define Hamiltonian path.

(d) What is the number of arrangements of all the six letters in the word PEPPER?

(e) What is a tautology?

(f) Give the axiomatic definition of probability.

(g) What is curve fitting?

(h) What are existential and universal quantifiers?

(i) State the condition for convergence of Gauss–Jacobi method.

(j) When is a grammar said to be ambiguous?

(k) State Cook's theorem.

(l) Find the number of ways that a party of seven persons can arrange themselves :

(i) in a row of seven chairs

(ii) around a circular table.

(m) Define a K-connected graph.

(n) Consider the language $L = \{a^n b^n \,;\, n \geq 0\}$. Find a context free grammar G which generates L. Find a regular grammar G which generates L.

(o) Distinguish between Mealy and Moore machines.

Group - A

(Discrete Mathematical Structures)

2. (a) Differentiate between edge-disjoint and vertex-disjoint sub graphs.

(b) What is the objective of Floyd's algorithm in graph theory?

(c) Illustrate the working principle of Floyd's algorithm with a suitable example.

(d) Differentiate between adjacency matrix and the incidence matrix. 3+1+8+4

3. (a) Prove that the number of internal vertices in a binary tree is one less than the number of pendant vertices.

(b) Prove that a simple graph with n vertices and k components have atmost $(n-k)(n-k+1)/2$ edges.

(c) If three dice is rolled, what is the probability that exactly two of the faces show a number less than or equal to 5?

(d) Find the number (m) of ways that nine toys can be divided among four children, if the youngest child is to receive three toys and each of the others two toys each. 4+5+4+3

4. (a) State and prove the generalised principle of Inclusion and Exclusion.

(b) A committee of 5 principals is to form from a group of 6 male principals and 8 female principals. If the selection is made randomly, find the probability that there are 3 female principals and 2 male principals.

(c) Explain big-Oh with a suitable diagram.

(d) Find the generating function for the Fibonacci series. 4+4+4+4

5. (a) Determine whether or not the given pair of well formed propositions are logically equivalent.

(i) $(x \rightarrow y)$ and $(\sim y \rightarrow \sim x)$

(ii) $((A \rightarrow B) \rightarrow C)$ and $(A \rightarrow (B \rightarrow C))$.

(b) State Bayes' theorem on conditional probability.

(c) Define the expected value and variance of a random variable over a given sample space.

(d) Solve the recurrence relation

$a_n = a_{n-1} + 2a_{n-2}$

with $a_0 = 2$ and $a_1 = 7$. 4+4+4+4

Group-B

(Numerical Methods and Algorithm)

6. (a) Solve the system of linear equations using Gauss-Seidel method up to two significant figures :

$$x_1 - 4x_2 + 10x_3 = 23$$
$$3x_1 + 10x_2 + x_3 = 17$$
$$20x_1 + 5x_2 - 2x_3 = 14$$

(b) Evaluate $\int_0^{\prime} \cos x \, dx$, taking 5 intervals. Clearly specify the method used.

(c) Find the relative percentage error in the approximate representation of $^4/_3$ by $1{\cdot}33$ 8+6+2

7. (a) Use 4th order $R-K$ method to solve the following differential equation correct up to five decimal places. Compute $y\,(0.3)$ from

$$\frac{dy}{dx} = x + y,\ y(0) = 1,\ \ h = 0.1\,.$$

(b) Write an algorithm for the Newton Raphson method to find the roots of a real valued function $f\,(x) = 0$. 8+8

Group-C

(Formal Languages and Automata Theory)

8. (a) Obtain a grammar to generate the following over $\{a, b\}$:

(i) Set of all strings with exactly one a.

(ii) Set of all strings with at least one a.

(b) Construct an NFA for $r = (a + bb)^*ba^*$. Show the steps clearly.

(c) Give the formal definition of Turing machine. 4+8+4

9. (a) What do you mean by regular expression?

(b) Show that if L_1 is regular and L_2 is regular, then $L_1 \cap L_2$ is also regular.

(c) Consider a grammar G whose productions are $\mathrm{S} \rightarrow \mathrm{aAS} \mid \mathrm{a},\ \mathrm{A} \rightarrow \mathrm{SbA} \mid \mathrm{SS} \mid \mathrm{ba}$.

Show that $\mathrm{S} \overset{*}{\Rightarrow} \mathrm{aabbaa}$ and construct a derivation tree whose yield is aabbaa.

(d) Is it possible for a regular grammar to be ambiguous? 2+4+6+4